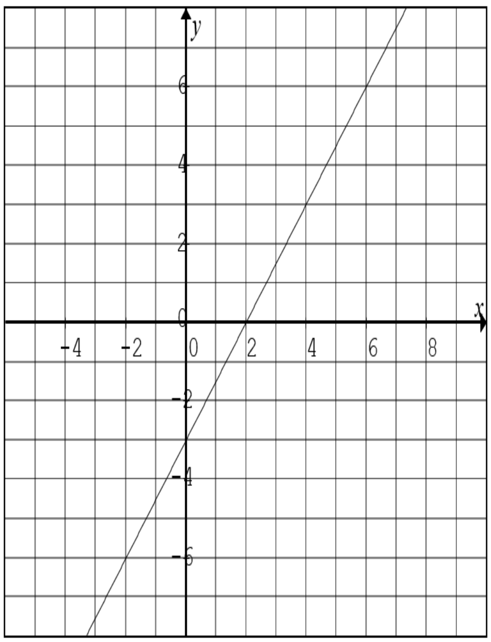
Let's do a quick review of linear functions from Pre-calc 10. The following diagram shows the graph of $3x-2y=6$.




- All linear functions can be expressed in standard form: $y=mx+b$
- $m = slope = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
- $b =$ y-intercept $(0, y)$
- The degree of the equation is always 1
- Determine the equation of this function in standard form as well as the slope and y-intercept.

This year, we will look specifically at quadratic functions. They all have the following characteristics:

- The graph is "U"-shaped (aka, parabola)
- The equation in general form is: $y = Ax^2 + Bx + C$, where A, B, & C are real numbers
- The degree of the equation is always 2

Let's assume $A=1$, $B=0$, & $C=0$. If we substitute these numbers into the general form equation

$$y = Ax^2 + Bx + C$$

$$y = 1x^2 + 0x + 0$$ this will be our "standard" quadratic equation

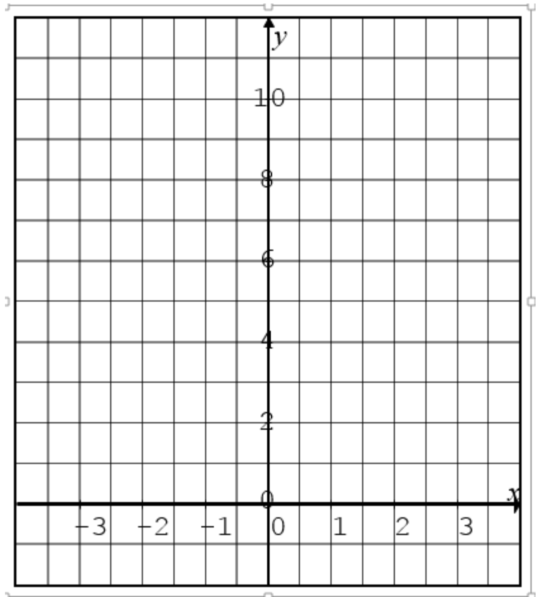
$$y = x^2$$

**Example 1**: Determine A, B, & C for each equation

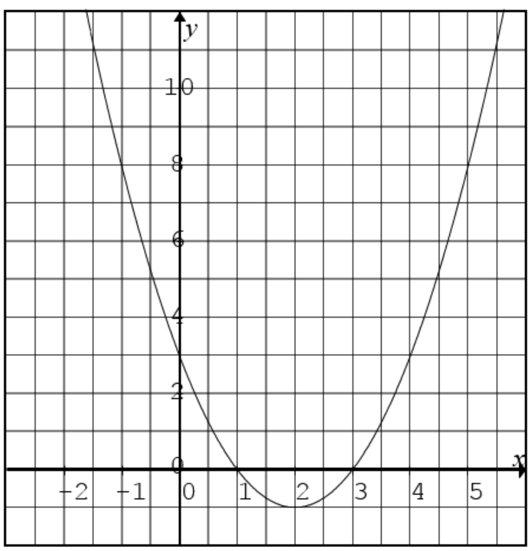
| | | |
|---|---|---|
| a) $y=3x^2+2x-5$ | b) $y=5x^2+17$ | c) $y=6x+12$ |

We're going to use a table of values and the "standard" equation $y = x^2$ to see why quadratic functions have parabolic, ie, "U"-shaped graphs.

| x | y |
|---|---|
| -3 | |
| -2 | |
| -1 | |
| 0 | |
| 1 | |
| 2 | |
| 3 | |




Let's look at specific terms and properties of quadratic functions using the equation $y = x^2 - 4x + 3$ and it corresponding graph.




- Vertex: The tip of the parabola (min or max) Our graph shows a min point @ $(2, -1)$
- Axis of Symmetry: A line cutting the graph in half, resulting in mirror images. Our graph shows an A of S @ $x = 2$
- X-intercepts: point(s) where the graph touches/crosses the horizontal axis. Our graph shows x-intercepts @ $x = 1$ & $x = 3$ or @ $(1, 0)$ & $(3, 0)$
- Y-intercept: point where the graph crosses the vertical axis. Our graph has a y-intercept @ $y = 3$ or @ $(0, 3)$

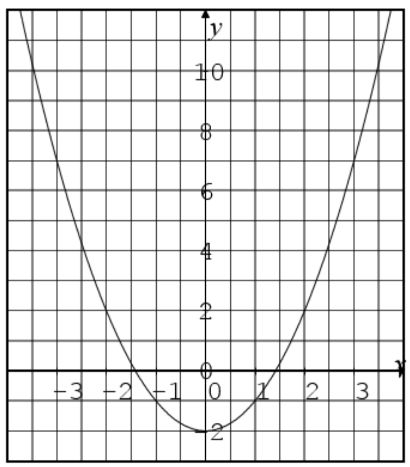
These four terms will be very important for you to understand.

Don't forget the following when determining your intercepts:

- To calculate the x-intercepts, remember the y-coordinate = 0
- To calculate the y-intercepts, remember the x-coordinate = 0

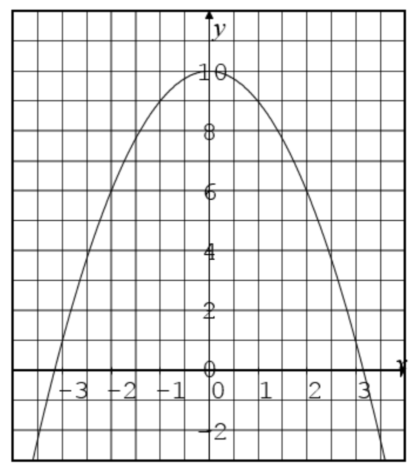
If we look at 2 quadratic functions with the equations $y = x^2 - 2$ & $y = -x^2 + 10$ and their corresponding graphs, please take notice of a few more things.

$y = x^2 - 2$



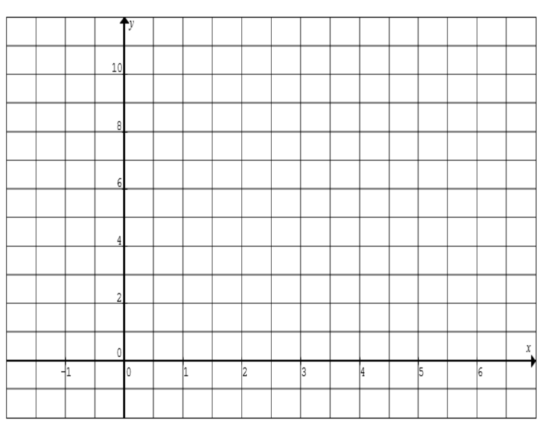

- The graph "opens up"
- The vertex $(0,-2)$ is a "minimum pt."
- The domain is $x \in \mathbb{R}$
- The range is all values above the vertex, i.e., $y \geq -2$

$y = -x^2 + 10$




- The graph "opens down"
- The vertex $(0,10)$ is a "maximum pt."
- The domain is $x \in \mathbb{R}$
- The range is all values below the vertex, i.e., $y \leq 10$

**Example 2**: Graph the function $y = x^2 - 5x + 6$ and answer the following questions.




| x | -1 | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|---|
| y | | | | | | | | |

a) Vertex:

b) A of S:

c) x-intercepts:

d) y-intercept:

e) Domain:

f) Range:

It will be especially important for us to be able to express quadratic functions in Standard Form.

- Using a table of values will enable you to draw the graph, but it takes too long and is inefficient
- The standard form for a Quadratic Function is: $\boxed{y=a(x-p)^2+q}$
- These are the following characteristics:

| | |
|---|---|
| 1. Vertex: $(p,q)$<br>2. A of S: $x=p$<br>3. x-intercept: make $y=0$ and solve for $x$ | 4. y-intercept: make $x=0$ and solve for $y$<br>5. Domain: $x\in\mathbb{R}$<br>6. Range: $y\le q$ or $y\ge q$ |

**Example 3**: Determine $a$, $p$, & $q$, vertex, and A of S for each equation.

| | | |
|---|---|---|
| a) $y=(x-3)^2+4$ | b) $y=-\frac{1}{2}(x+4)^2-11$ | c) $y=12-6(x+9)^2$ |

The '$a$' value in $y=ax^2$ serves two functions:

1. It will determine whether the graph opens up or down, i.e., is there a reflection about the horizontal axis
   - If $a=(+)ve$, the graph will open up
   - If $a=(-)ve$, the graph will open down, i.e., reflected about the x-axis
2. It will determine whether the graph is tall & thin or short & fat, i.e., is there a vertical expansion/compression
   - If $a\ge 1$, the graph will become tall & thin, i.e., vertical expansion
   - If $0\le a\le 1$, the graph will become short & fat, i.e., vertical compression

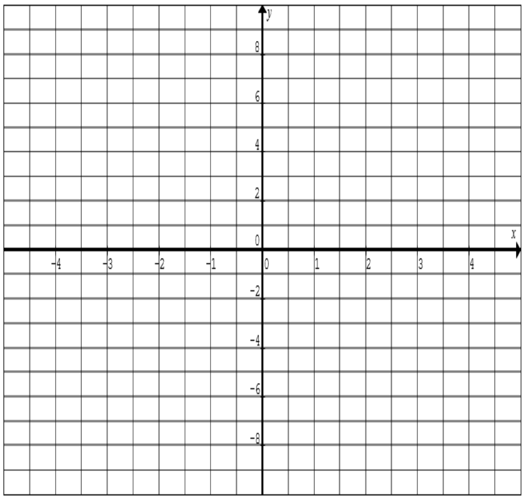
The most important fact is that only the y-coordinates are affected through multiplication

- Any vertical expansion or compression will change the shape of the graph
  - Shape of new graph will NOT BE CONGRUENT to original graph

- Any vertical reflection will not change the shape of the graph
    - Shape of new graph will BE CONGRUENT to original graph

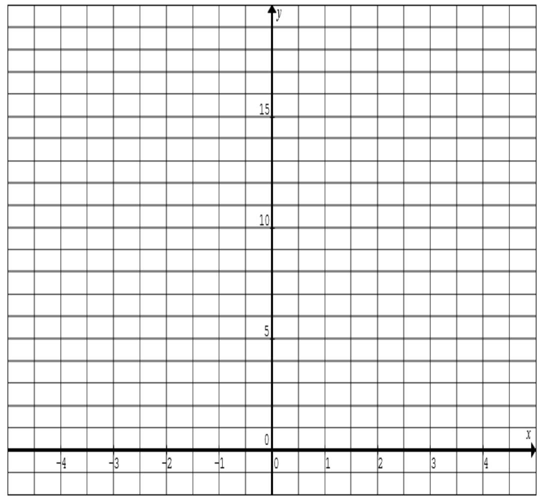
**Example 4**: Graph the following quadratic functions.

a) Given $y = x^2$, graph $y = -x^2$



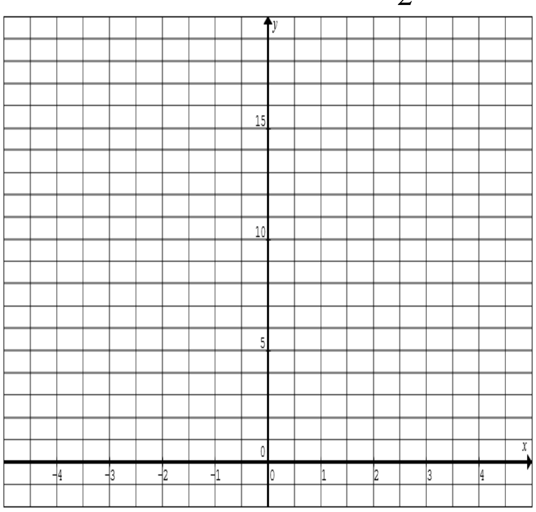

| x | | | | | | | |
|---|---|---|---|---|---|---|---|
| y | | | | | | | |
| y' | | | | | | | |

b) Given $y = x^2$, graph $y = 2x^2$



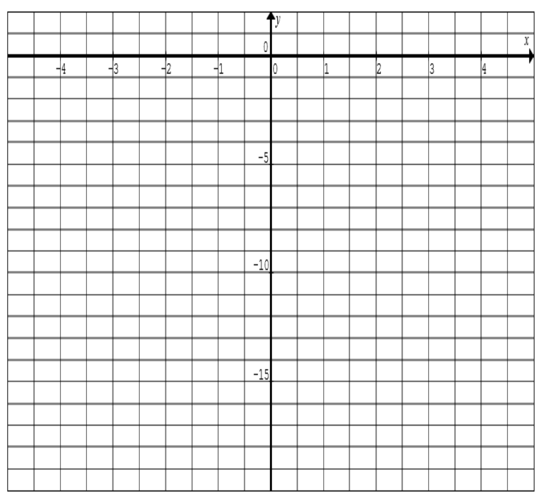

| x | | | | | | | |
|---|---|---|---|---|---|---|---|
| y | | | | | | | |
| y' | | | | | | | |

c) Given $y = x^2$, graph $y = \frac{1}{2}x^2$



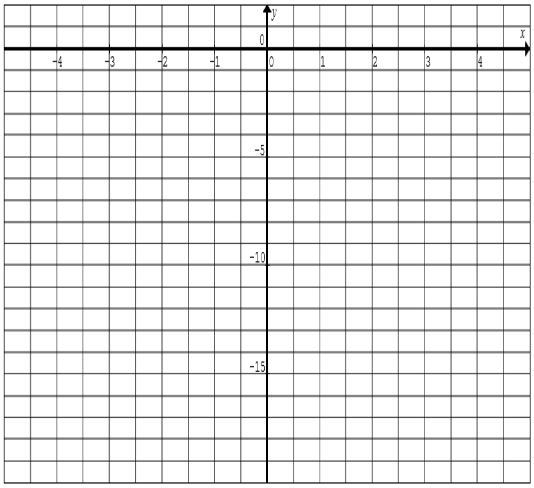

| x | | | | | | | |
|---|---|---|---|---|---|---|---|
| y | | | | | | | |
| y' | | | | | | | |

d) Given $y = -x^2$, graph $y = -4x^2$




| x | | | | | | | |
|---|---|---|---|---|---|---|---|
| y | | | | | | | |
| y' | | | | | | | |

e) Given $y = -x^2$, graph $y = -\frac{1}{3}x^2$




| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | | | | | | | |
| y | | | | | | | |
| y′ | | | | | | | |