SOL HW 4.2

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Name:_________________________ **Date:**_________________________

Math 9 Honours: Section 4.2 Binomial Expansions:

1. Expand the following expressions using Pascal's Triangle

a) $(x+a)^3 =$

$$\begin{array}{cccc} 1 & 3 & 3 & 1 \\ x^3 & x^2 & x & \\ & a & a^2 & a^3 \end{array}$$

$= x^3 + 3ax^2 + 3a^2x + a^3$

b) $(x-a)^4 =$

$$\begin{array}{ccccc} 1 & 4 & 6 & 4 & 1 \\ x^4 & x^3 & x^2 & x & \\ & (-a)^1 & (-a)^2 & (-a)^3 & (-a)^4 \end{array}$$

$= x^4 - 4ax^3 + 6a^2x^2 - 4a^3x + a^4$ //

c) $(x+a)^5 =$

$$\begin{array}{cccccc} 1 & 5 & 10 & 10 & 5 & 1 \\ x^5 & x^4 & x^3 & x^2 & x^1 & x^0 \\ & a & a^2 & a^3 & a^4 & a^5 \end{array}$$

$= x^5 + 5ax^4 + 10a^2x^3 + 10a^3x^2 + 5a^4x + a^5$ //

d) $(\sqrt{3}+a)^6 =$

$$\begin{array}{ccccccc} 1 & 6 & 15 & 20 & 15 & 6 & 1 \\ a^6 & a^5 & a^4 & a^3 & a^2 & a^1 & 1 \\ 1 & \sqrt{3} & 3 & 3\sqrt{3} & 9 & 9\sqrt{3} & 27 \end{array}$$

$= a^6 + 6\sqrt{3}a^5 + 45a^4 + 60\sqrt{3}a^3 + 135a^2 + 54\sqrt{3}a + 27$ //

e) $\left(2-\frac{1}{a}\right)^3 =$

$$\begin{array}{cccc} 1 & 3 & 3 & 1 \\ 8 & 4 & 2 & 1 \\ 1 & \left(-\frac{1}{a}\right) & \left(\frac{1}{a^2}\right) & \left(-\frac{1}{a^3}\right) \end{array}$$

$= 8 - \frac{12}{a} + \frac{6}{a^2} - \frac{1}{a^3}$

f) $(\sqrt{2}-1)^5$ (Expand and find the exact value without a calculator)

$$\begin{array}{cccccc} 1 & 5 & 10 & 10 & 5 & 1 \\ +4\sqrt{2} & -4 & +2\sqrt{2} & -2 & +\sqrt{2} & -1 \end{array}$$

$4\sqrt{2} - 20 + 20\sqrt{2} - 20 + 5\sqrt{2} - 1$

$= \boxed{29\sqrt{2} - 36}$

2. Expand the following expression and indicate what the constant term is:

i) $\left(x^2+\frac{1}{x}\right)^5 =$

$$\begin{array}{cccccc} 1 & 5 & 10 & 10 & 5 & 1 \\ (x^2)^5 & (x^2)^4 & (x^2)^3 & (x^2)^2 & (x^2)^1 & 1 \\ 1 & \frac{1}{x} & \left(\frac{1}{x}\right)^2 & \left(\frac{1}{x}\right)^3 & \left(\frac{1}{x}\right)^4 & \left(\frac{1}{x}\right)^5 \end{array}$$

$= x^{10} + 5x^7 + 10x^5 + 10x + \frac{5}{x^2} + \frac{1}{x^5}$ [No Constant Term]

ii) $\left(2x^3-\frac{1}{x^2}\right)^{10} =$

$$\begin{array}{ccccccccccc} 1 & 10 & 45 & 120 & 840 & 1512 & \boxed{840} & 120 & 45 & 10 & 1 \\ (2x^3)^{10} & (2x^3)^9 & (2x^3)^8 & (2x^3)^7 & (2x^3)^6 & (2x^3)^5 & \boxed{(2x^3)^4} & (2x^3)^3 & (2x^3)^2 & (2x^3)^1 & (2x^3)^0 \\ 1 & \frac{1}{x^2} & \frac{1}{x^4} & \frac{1}{x^6} & \frac{1}{x^8} & \frac{1}{x^{10}} & \boxed{\frac{1}{x^{12}}} & \frac{1}{x^{14}} & \frac{1}{x^{16}} & \frac{1}{x^{18}} & \frac{1}{x^{20}} \end{array}$$

$840\,(2x^3)^4 \cdot \frac{1}{x^2}$

$= 840 \cdot 16$

$= 13440$ //

iii) $\left(x^4-\frac{2}{x}\right)^4$

$$\begin{array}{ccccc} 1 & 4 & 6 & 4 & 1 \\ (x^4)^4 & (x^4)^3 & (x^4)^2 & (x^4)^1 & (x^4)^0 \\ \left(\frac{-2}{x}\right)^0 & \left(\frac{-2}{x}\right)^1 & \left(\frac{-2}{x}\right)^2 & \left(\frac{-2}{x}\right)^3 & \left(\frac{-2}{x}\right)^4 \end{array}$$

No Constant Term!

3. Suppose that the number "x" satisfies the equation: $5 = x + x^{-1}$. What is the value of $x^4 + x^{-4}$?

$\left(x+\frac{1}{x}\right)^2=(5)^2$ $\qquad x^4+\frac{1}{x^4}=?$

$\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}\right)=25$

$x^2+1+1+\frac{1}{x^2}=25$

$\boxed{x^2+\frac{1}{x^2}=23}$

$\left(x^2+\frac{1}{x^2}\right)\left(x^2+\frac{1}{x^2}\right)=529$

$x^4+1+1+\frac{1}{x^4}=529$

$x^4+\frac{1}{x^4}=527$

4. There are integers "a" and "b" such that: $\left(1+\sqrt{2}\right)^{16} = a + b\sqrt{2}$. What is the value of "a"?

① FOIL THINGS TO MAKE THINGS SIMPLER:

$(1+\sqrt2)^2=(1+\sqrt2)(1+\sqrt2)$

$=1+2\sqrt2+2$

$=3+2\sqrt2$

So $(1+\sqrt2)^{16}=(3+2\sqrt2)^8=(17+12\sqrt2)^4$

② $(3+2\sqrt2)(3+2\sqrt2)$

$=9+12\sqrt2+8$

$=17+12\sqrt2$

③ $(1+\sqrt2)^{16}=(17+12\sqrt2)^4$

$= \begin{array}{ccccc} 1 & 4 & 6 & 4 & 1 \\ (17)^4 & (17)^3 & (17)^2 & (17) & 1 \\ 1 & (12\sqrt2) & (12\sqrt2)^2 & (12\sqrt2)^3 & (12\sqrt2)^4 \end{array}$

$=17^4+4\cdot17^3\cdot12\sqrt2+6\cdot17^2\cdot144(2)+68\cdot12^3\cdot2\sqrt2+12^4(4)$

$a=17^4+6\cdot17^2\cdot288+12^4(4)$ $\qquad b\sqrt2=4\cdot17^3\cdot12\sqrt2+68\cdot12^3\cdot2\sqrt2$

5. Given the expression: $(x+a)^5$, what are the values of "a" and "x" if the third term of the expression is equal to 4320 and the fifth term is equal to 3840.

6. Given the expression: $(a-b)^4$, what are the values of "a" and "b" if the third term of the expression is equal to 108 and the fourth term is $-24\sqrt{2}$.

$(a-b)^4= \begin{array}{ccccc} 1 & 4 & 6 & 4 & 1 \\ a^4 & a^3 & a^2 & a^1 & a^0 \\ (-b)^0 & (-b)^1 & (-b)^2 & (-b)^3 & (-b)^4 \end{array}$

$a^4-4a^3b+\underbrace{6a^2b^2}_{108}-\underbrace{4ab^3}_{-24\sqrt2}+b^4$

$6a^2b^2=108$ $\qquad 4ab^3=24\sqrt2$

$a^2b^2=18$ $\qquad \boxed{ab^3=6\sqrt2}$

$\boxed{ab=\pm3\sqrt2}$

$\boxed{a=\pm3}$

$\frac{\cancel{a}b^3}{\cancel{a}b}=\frac{6\cancel{\sqrt2}}{\pm3\cancel{\sqrt2}}$

$b^2=2$

$\boxed{b=\pm\sqrt2}$

a, b

$(3,\sqrt2)$

$(-3,-\sqrt2)$

7. If the first term of the expansion $(a+b)^6$ is equal to 512 and the last term is 5832, then what is the value of the expression?

8. If the sum of the coefficients of $(a+b)^n$ is equal to 128, how many terms are in the expansion?

$1 = 2^0$
$1 \quad 1 = 2^1$
$1 \quad 2 \quad 1 = 2^2$
$1 \quad 3 \quad 3 \quad 1 = 2^3$
$1 \quad 4 \quad 6 \quad 4 \quad 1 = 2^4$

$2^7 = 128$

$\therefore$ 8 terms //

9. If $(x+3)^5 - (x+2)^4 = Ax^5 + Bx^4 + Cx^3 + Dx^2 + Ex + F$. What is the value of $A+B+C+D+E+F$?

10. The 4th term of $(x-0.5)^n$ is $-15x^7$. What is the value of "n"?

$\begin{array}{cccc} 1 & n & nC_2 & nC_3 \\ x^n & x^{n-1} & x^{n-2} & x^{n-3} \\ (-0.5)^0 & (-0.5)^1 & (-0.5)^2 & (-0.5)^3 \end{array}$

$= -15x^7$

$\therefore \boxed{n=10}$

$10C_3 \cdot x^7 \cdot (-0.5)^3 =$

11. The 7th term of $(2x-1)^n$ is $112x^2$. What is the value of "n"?

12. Determine "b" such that $(x-b)^{10}$ has the term $-1875x^7$

$\begin{array}{cccc} 10C_0 & 10C_1 & 10C_2 & 10C_3 \\ x^{10} & x^9 & x^8 & x^7 \\ b^0 & (-b)^1 & (-b)^2 & (-b)^3 \end{array}$

$= -1875x^7$

$10C_3 \cdot x^7 (-b)^3 = -1875$

$120 \cdot (-b)^3 = -1875$

$\frac{120(-1)b^3}{120(-1)} = \frac{-1875}{-120}$

$b = \sqrt[3]{\frac{-1875}{-120}}$

$\boxed{b = \frac{5}{2}}$ //

13. $\left(\sqrt{3}-1\right)^4-\left(2\sqrt{3}+2\right)^3=A+B\sqrt{C}$. What is the value of $A+B+C$?

$(\sqrt{3}-1)^4 =$ 1 4 6 4 1

$+1 - \sqrt{3} + 3 - 3\sqrt{3} + 9$

$= 1 - 4\sqrt{3} + 18 - 12\sqrt{3} + 9$

$(2\sqrt{3}+2)^3 = 2^3(\sqrt{3}+1)^3$

$= 8(\sqrt{3}+1)^3 = 8($ 1 3 3 1 $)$

$\sqrt{3}$ 3 $3\sqrt{3}$

$= 8(1 + 3\sqrt{3} + 9 + 3\sqrt{3})$

$= 8 + 24\sqrt{3} + 72 + 24\sqrt{3}$

$=$ ______

14. Solve for all the possible value(s) of "x" $\frac{x^3+3x^2+3x+1}{x+1}=25$

$\frac{(x+1)^3}{x+1} = 25$

$(x+1)^2 = 25$

$x+1=5$ $x+1=-5$

$\boxed{x=4}$ $\boxed{x=-6}$

15. Solve for all the possible value(s) of "x" $\frac{x^5-5x^4+10x^3-10x^2+5x-1}{(1-x)^2}=343$

16. The expansion of $(3+2x)(1-x)^n=ax^2-10x+3$ Find the values of "a" and "n"

$(3+2x)(1-x)$

$3 - 3x + 2x - 2x^2$

$(3 - x - 2x^2)(1-x)$

$3 - x - 2x^2 - 3x + x^2 + 2x^3$

$2x^3 - x^2 - 4x + 3 =$

$(3+2x)(1-x)^3$

$(2x^3 - x^2 - 4x + 3)(1-x)$

$2x^3 - x^2 - 4x + 3 - 2x^4 + x^3 + 4x^2 - 3x$

$-2x^4 + 3x^3 + 3x^2 - 7x + 3$

$(3+2x)(1-x)^4$

$(-2x^4 + 3x^3 + 3x^2 - 7x + 3)(1-x)$

$-2x^4 + 3x^3 + 3x^2 - 7x + 3$

$+2x^5 - 3x^4 - 3x^3 + 7x^2 - 3x$

$\boxed{2x^5 - 5x^4 + 10x^2} \boxed{-10x} \boxed{+3}$

$\boxed{ax^2} - \boxed{10x} \boxed{+3}$

$\boxed{n=4}$ $ax^2 = 2x^5 - 5x^4 + 10x^2$

$\boxed{a = 2x^3 - 5x^2 + 10}$

17. Challenge: Find the coefficient of x^2 in the expression: $\left(2+x-x^2\right)^4-\left(3-2x-x^2\right)^5$

$= (2 + x - x^2)^4$

$= (-x^2 + x + 2)^4$

$= (-1)^4 (x^2 - x - 2)^4$

$= (x^2 - x - 2)^4$

$= [x^2 - (x+2)]^4$

$= [(x-2)(x+1)]^4$

$(3 - 2x - x^2)^5$

$(-x^2 - 2x + 3)^5$

$(-1)^5 (x^2 + 2x - 3)^5$

$-1 (x^2 + (2x-3))^5$

$-1 ((x-1)(x+3))^5$

$[x^2 - (x+2)]^4 + [x^2 + (2x-3)]^5$

1 4 6 4 1

$(x^2)^4 - (x^2)^3 \quad (x^2)^2 \quad (x^2)^1$

$(x+2)^1 + (x+2)^2 - (x+2)^3 + (x+2)^4$

$\underset{N}{x^8} - 4x^6\underset{N}{(x+2)} + 6x^4\underset{N}{(x+2)^2} - 4x^2\underset{Y}{(x+2)^3} + \underset{Y}{(x+2)^4}$