SOL HW 3.3

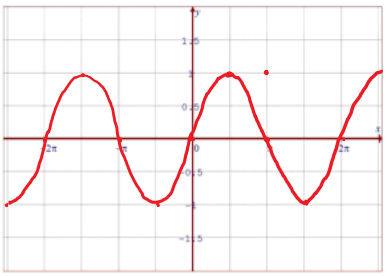
January 17, 2017 1:25 PM

Name: Key Date:

Section 3.3 Graphing Sine Cosine and Tangent Functions

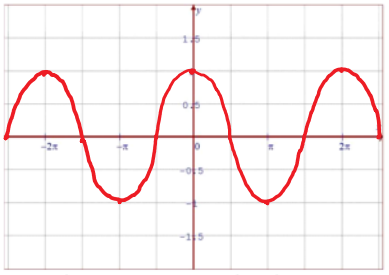
1. Graph the following function for $-2\pi \le \theta \le 2\pi$. Indicate the Period, Amplitude, Domain, and Range:

i) $y = \sin x$



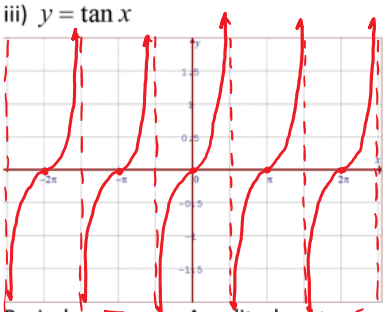
Period: 2π Amplitude: 1
Domain: $x \in \mathbb{R}$ Range: $-1 \le y \le 1$

ii) $y = \cos x$



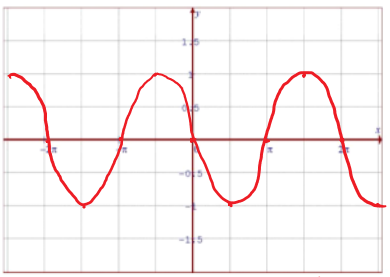
Period: 2π Amplitude: 1
Domain: $x \in \mathbb{R}$ Range: $-1 \le y \le 1$

iii) $y = \tan x$



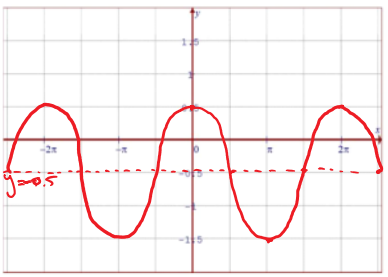
Period: π Amplitude: none
Domain: $x \ne n\pi$ Range: $y \in \mathbb{R}$

iv) $y = -\sin x$



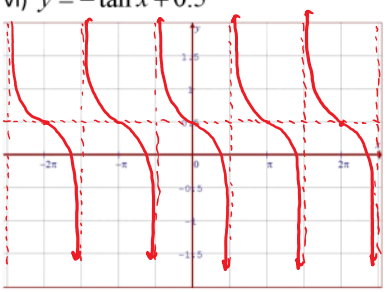
Period: 2π Amplitude: 1
Domain: $x \in \mathbb{R}$ Range: $-1 \le y \le 1$

v) $y = -\cos x - 0.5$



Period: 2π Amplitude: 1
Domain: $x \in \mathbb{R}$ Range: $-1.5 \le y \le 0.5$

vi) $y = -\tan x + 0.5$



Period: π Amplitude: none
Domain: $x \ne n\pi$ Range: $y \in \mathbb{R}$

2. Indicate the general formula for the vertical asymptotes of $y = \tan x$

① $y = \tan x = \frac{\sin x}{\cos x}$ ② V.A.

THE V.A. occur when $\cos x = 0$. $x = n \times \pi$, where $n \in \pm 1, 2, 3, 4, 5, \ldots$

3. When the graph of $y = \sin x$ and $y = \cos x$ are drawn on the same graph for $0 < x < 2\pi$ in which quadrants do they intersect? What are the coordinates of the points of intersection?

① when $\sin x = \cos x$, divide both sides by $\cos x$.

$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\tan x = 1$

② since $\tan x = 1$, then it must be in Q1, Q3

S	A
T	C

$\tan x = 1$

$x = \tan^{-1}(1)$

$x = 45°, 225°$

$= \frac{\pi}{4}, \frac{5\pi}{4}$

③ Coordinates

$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$.

$\therefore \left(\frac{\pi}{4}, \frac{1}{\sqrt{2}}\right) \therefore \left(\frac{5\pi}{4}, \frac{1}{\sqrt{2}}\right)$

$\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right) \therefore \left(\frac{5\pi}{4}, \frac{\sqrt{2}}{2}\right)$ or

4. Given that $\sin\theta > 0$ and $\cos\theta < 0$, what is the range of possible values of θ if $0 < \theta < 2\pi$?

① $\sin\theta > 0$ so θ must be in Q1 or Q2.

S	A
T	C

② $\cos < 0$ so θ must be in Q2.

③ Range of values for θ

$90° < \theta < 180°$

5. Indicate TRUE or FALSE: $\sin\theta > 0$ and $\cos\theta > 0$, then $\tan\theta$ can be either positive or negative.

① since $\sin\theta > 0$ then θ must be in Q1 or Q2.

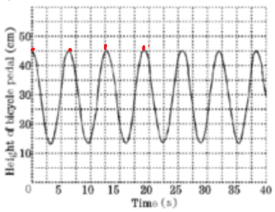
② since $\cos\theta > 0$, then θ must be in Q1.

③ since θ must be in Q1, then $\tan\theta$ can only be positive

"FALSE"

6. Given each of the following trigonometric graphs, indicate the amplitude and period

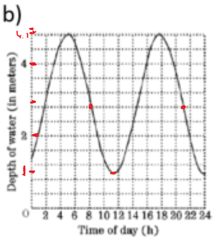
a)



$\frac{45-12}{2} = \frac{33}{2}$

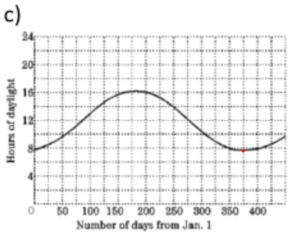
Period= ≈7s. Amplitude= ≈16.5cm

b)



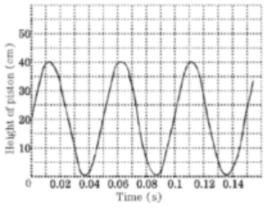
Period= 13h. Amplitude= 3.7m

c)



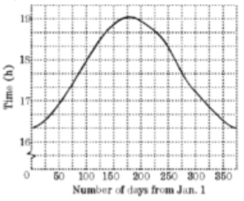
Period= 375 D. Amplitude= 4H.

d)



Period= 0.05s Amplitude= 20cm

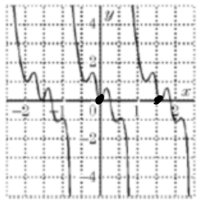
e)



$19 - 16\frac{1}{3}$

Period= 375 Amplitude= $\frac{4}{3}$ h.

f)



Period= 1.5 Amplitude= NONE

7. If How do the intersections of $y = \sin x$ and $y = \cos x$ relate with the graph of $y = \tan x$?

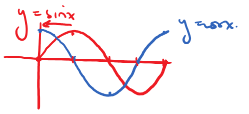
① The intersection of $y = \sin x$ & $y = \cos x$ is when $\sin x = \cos x$

$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\tan x = 1$

So, when $\tan x = 1$,

8. How many units should the graph of $y = \sin x$ be shifted horizontally so that it will overlap the graph of $y = \cos x$?



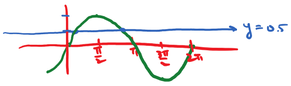
① The sine graph will need to be shifted a quarter of it's period to overlap the cosine graph.

② $p = 2\pi$

$\frac{2\pi}{4} = \frac{\pi}{2}$

9. When the graph of $y = \sin x$ and $y = 0.5$ are drawn on the same graph for $0 < x < 2\pi$ in which quadrants do they intersect? What are the coordinates of the points of intersection?

① When $\sin x = 0.5$, since the ratio is positive, it must be in Q1 & Q2.



② $\sin x = 0.5$

$x = \sin^{-1}(0.5)$

$x = 30°, 150°$

or

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

③ Coordinates

$\left(\frac{\pi}{6}, 0.5\right)$ & $\left(\frac{5\pi}{6}, 0.5\right)$

Q1 Q2.

10. What is the amplitude and period of the graph $y = A\sin(Bx)$ if $A = -3$ and $B = 2$?

① The amplitude will be the number multiplied to $\sin x$.

$y = (\text{Amp}) \times \sin x$

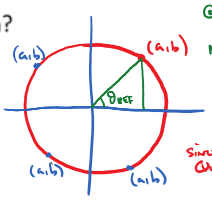
since $A = -3$, the amplitude $= |-3| = 3$

② Period will be 2π divided by the coefficient of x.

period $= \frac{2\pi}{B} = \frac{2\pi}{2} = \pi$

11. Given that the terminal arm intersects the unit circle at coordinates (a,b), what is the reference angle and the angle in standard position?

① For any point (a,b) in an unit circle, 'a' is the x coord, and 'b' is the y coord.



② θ_{ref} must always be positive, so it doesn't matter which quadrant (a,b) is in.

$\tan\theta = \frac{b}{a}$

$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

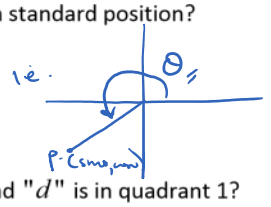
Since (a,b) can be in other quadrants, then:

$\theta = \tan^{-1}\left(\left|\frac{b}{a}\right|\right)$

Since the ratio of $\frac{b}{a}$ can be negative, take the abs. value so θ' will always be between 0 & 90°.

12. If point "P" is on the unit circle with coordinates defined by $(\sin\theta, \cos\theta)$, what is θ in standard position?

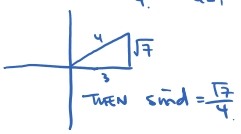
"θ" IS STANDARD POSITION WITH BE AN ANGLE CREATED BY THE TERMINAL ARM ROTATING FROM THE POSITIVE x-axis TO THE POINT $P(\sin\theta, \cos\theta)$



13. Given the identity $\sin 2a = 2\sin a \times \cos a$, what is the value of $\sin 2d$ if $\cos d = \frac{3}{4}$ and "d" is in quadrant 1?

Find the exact value.

① IF $\cos d = \frac{3}{4}$ IN Q1,



THEN $\sin d = \frac{\sqrt{7}}{4}$

② $\sin 2d = 2\sin d \cos d$

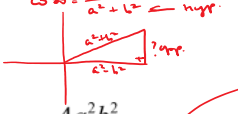
$= 2\left(\frac{\sqrt{7}}{4}\right) \times \frac{3}{4}$

$= \frac{6\sqrt{7}}{16} = \frac{3\sqrt{7}}{8}$

14. If $\cos\theta = \frac{a^2 - b^2}{a^2 + b^2}$ and $0° \le \theta \le 90°$, find the value of $\sin\theta$:

a) $\frac{2ab}{a^2 + b^2}$ b) $\frac{4ab}{a^2 + b^2}$ c) $\frac{2a^2b^2}{a^2 + b^2}$ d) $\frac{4a^2b^2}{a^2 + b^2}$ e) $\frac{a^2b^2}{2a^2 + 2b^2}$

① $\cos\theta = \frac{a^2 - b^2}{a^2 + b^2}$ ← adj / ← hyp



② $opp^2 = (a^2+b^2)^2 - (a^2-b^2)^2$

$opp^2 = (a^2+b^2+a^2-b^2)(a^2+b^2-a^2+b^2)$

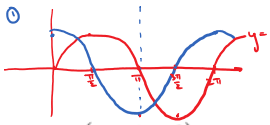
$opp^2 = (2a^2)(2b^2)$

$opp = 2ab$

③ $\sin\theta = \frac{2ab}{a^2+b^2}$ A

15. If $0° \le \theta \le 180°$ and $\sin\theta \ge \cos\theta$, then:

a) $0 \le \theta \le 45°$ b) $45° \le \theta \le 90°$ c) $45° \le \theta \le 180°$ d) $90° \le \theta \le 180°$ e) $0 \le \theta \le 90°$



② WE ARE LOOKING FOR PARTS OF THE GRAPH WHERE THE SINE GRAPH IS ABOVE THE COSINE GRAPH.

③ FOR $0 \le \theta \le 180°$, THE TWO GRAPHS INTERSECT AT 45°

∴ $45° \le \theta \le 180°$

16. $\cos(270° - \theta) =$

a) $-\cos\theta$ b) $\cos\theta$ c) $-\sin\theta$ d) $\sin\theta$ e) $\sin\theta\cos\theta$

$\cos(a-b) = \cos a\cos b + \sin a \sin b$

$\cos(270° - \theta) = \cos 270°\cos\theta + \sin 270°\sin\theta$

$= 0 \times \cos\theta + (-1)\sin\theta$

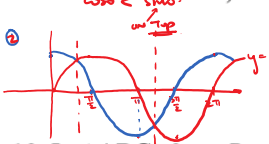
$= -\sin\theta$

17. If $\sin 2a < 0$, $\cos a - \sin a < 0$, which quadrant is angle a in?

a) I b) II c) III d) IV

① $\cos\theta - \sin\theta < 0$

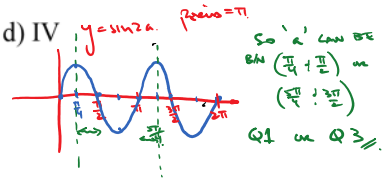
$\cos\theta < \sin\theta$

② 

③ THE TWO GRAPHS INTERSECT AT 45° & 225°, SO THE ANGLE MUST BE IN BETWEEN THAT TWO ANGLES.

④ $y = \sin 2a$

WITH A '2' IN FRONT OF THE ANGLE, THE GRAPH IS COMPRESSED IN HALF.



So 'a' CAN BE BTW $\left(\frac{\pi}{4} + \frac{\pi}{2}\right)$ OR $\left(\frac{5\pi}{4} + \frac{3\pi}{2}\right)$

Q1 OR Q3

18. In ΔABC, $2\cos B\cos A = \sin C$. What kind of shape is the triangle?

a) Right triangle

SINCE 45°, 45°, 90° IS A RIGHT TRIANGLE, THEN YES. HOWEVER, IT WON'T WORK WITH ANY KIND OF RIGHT TRIANGLE.

b) Equilateral triangle

$A = B = C = 60°$

$\sin 60° \stackrel{?}{=} 2\cos 60°\cos 60°$

$\frac{\sqrt{3}}{2} \stackrel{?}{=} 2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

c) 45-45-90 triangle

$A = 90° \quad B = C = 45°$

$2\cos 45°\cos 90° \stackrel{?}{=} \sin 45°$

$2\left(\frac{1}{\sqrt{2}}\right)(0) \stackrel{?}{=} \frac{1}{\sqrt{2}}$ ✗

$A = B = 45° \quad C = 90°$

$2\cos 45°\cos 45° = \sin 90°$

$2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = 1$ ✓

d) Isosceles triangle

SINCE 45°, 45°, 90° IS AN ISOSCELES TRIANGLE, THEN YES. HOWEVER, IT WON'T WORK WITH ANY ISOS. TRIANGLE.

19. $0 < \beta < 2\pi$ what does β need to be in order for $\sin\beta > \cos\beta$ to be true?

A. $\frac{\pi}{4} < \beta < \frac{\pi}{2}$ and $\pi < \beta < \frac{5}{4}\pi$

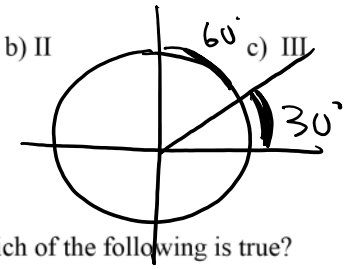
B. $\frac{\pi}{4} < \beta < \pi$

C. $\frac{\pi}{4} < \beta < \frac{5}{4}\pi$

D. $\frac{\pi}{4} < \beta < \pi$ *and* $\frac{5}{4}\pi < \beta < \frac{3}{2}\pi$

20. Angle A, B are both acute angles. Point P has coordinates (cosB-sinA, sinB-cosA) Which quadrant is point P in?

a) I b) II c) III d) IV



	1	2	3	4
∠A	30°	30°	60°	60°
∠B	30°	60°	30°	60°

21. sinα>sinβ Which of the following is true?

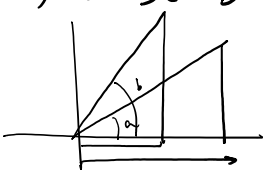
a) If α,β are in the quadrant I, then cosα>cosβa

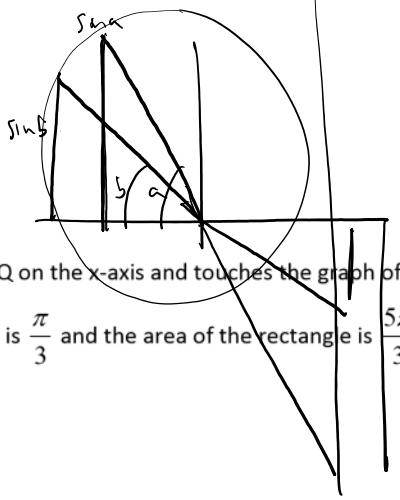
b) If α,β are in the quadrant II, then tanα>tanβ

c) If α,β are in the quadrant III, then cosα>cosβ

d) If α,β are in the quadrant IV, then tanα>tanβ

a) a=30° b=60°





22. A rectangle PQRS has side PQ on the x-axis and touches the graph of $y = k\cos(x)$ at the point "S" and "R" as shown. If the length of PQ is $\frac{\pi}{3}$ and the area of the rectangle is $\frac{5\pi}{3}$, what is the value of "k"?