SOL HW 3.1

May 11, 2018 11:03 AM

Name: Key Date:

Section 3.1 Radians and Angles in Standard Position

1. Convert the following angles to radians in terms of π. Show your work.

a) 60° $\frac{\pi}{3}$	b) 30° $\frac{\pi}{6}$	c) 150° $150\times\frac{\pi}{180}=\frac{5\pi}{6}$	d) 210° $\frac{7\pi}{6}$
e) 90° $\frac{\pi}{2}$	f) 135° $\frac{3\pi}{4}$	g) 225° $\frac{5\pi}{4}$	h) 240° $\frac{8\pi}{6}$
i) 315° $\frac{7\pi}{4}$	j) 360° 2π	k) 330° $\frac{11\pi}{6}$	l) 1050° $\frac{35\pi}{6}$

2. Convert the following to the nearest degree. Show your work.

a) $\frac{2\pi}{3}$ 120°	b) $\frac{2\pi}{6}$ 60°	c) $\frac{2\pi}{4}$ 90°	d) $\frac{5\pi}{3}$ 300°
e) $\frac{16\pi}{12}$ 240°	f) $\frac{11\pi}{3}$ 660°	g) $\frac{7\pi}{6}$ 210°	h) $\frac{15\pi}{4}$ 675°
i) $\frac{\pi}{12}$ 15°	j) $\frac{5\pi}{6}$ 150°	k) $\frac{3\pi}{20}$ 27°	l) $\frac{22\pi}{9}$ 440°

3. Determine the arc length that subtends each angle at the center of the circle with radius 10cm.

a) 60° $\theta=\frac{\pi}{3}$ $a=10\times\frac{\pi}{3}=\frac{10\pi}{3}$ cm	b) 150° $\theta=\frac{5\pi}{6}$ $a=r\times\theta=10\times\frac{5\pi}{6}=\frac{50\pi}{6}$ cm	c) 240° $\theta=\frac{4\pi}{3}$ $a=10\times\frac{4\pi}{3}=\frac{40\pi}{3}$ cm
d) $\frac{\pi}{12}$ $a=10\times\frac{\pi}{12}=\frac{5\pi}{6}$ cm	e) $\frac{5\pi}{3}$ $a=\frac{5\pi}{3}\times10=\frac{50\pi}{3}$ cm	f) $\frac{7\pi}{6}$ $a=10\times\frac{7\pi}{6}=\frac{35\pi}{3}$ cm

4. Graph each angle in standard position. Find the reference angle.

a) $\frac{2\pi}{3}$ $\theta_R=\frac{\pi}{3}$	b) $-\frac{10\pi}{6}=-\frac{5\pi}{3}$ $\theta_R=\frac{\pi}{3}$	c) $\frac{15\pi}{4}$ $\theta_R=\frac{\pi}{4}$	d) $-\frac{7\pi}{3}$ $\theta_R=\frac{\pi}{3}$
e) 800° $\theta=80^\circ$ $\theta_{ref}=80^\circ$	f) 1000° $\theta_{ref}=80^\circ$	g) -500° $\theta_{ref}=40^\circ$	h) -3000° $\frac{3000}{360}=8.333$ $\frac{1}{3}$ of a circle $=120^\circ$ $\theta_{ref}=60^\circ$

5. In what quadrants are the following angles in?

a) $35°$ Q1	b) $900°$ NEGATIVE x-axis	c) $\frac{5\pi}{4}$ Q3	d) $\frac{22\pi}{3} = 7\frac{1}{3}\pi$ Q3
e) $-475°$ Q3	f) $-2590°$ Q4	g) $\frac{-9\pi}{5}$ Q1	h) $\frac{-17\pi}{4}$ Q4

6. Evaluate each of the following trigonometric functions without a calculator:

a) $\sin 60°$ $= \frac{\sqrt{3}}{2}$	b) $\cos 90°$ 0	c) $\tan\frac{\pi}{2}$ UNDEFINED	d) $\cos\frac{\pi}{3}$ $\frac{1}{2}$
e) $\sin 45°$ $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	f) $\tan 30°$ $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	g) $\sin\frac{\pi}{6}$ $\frac{1}{2}$	h) $\tan 0°$ 0

7. What is the smallest positive coterminal angle of $2000°$?

① $\frac{2000}{360} = 5.5$ ② $2000 - 5(360) = 200°$

8. Give a general formula for all the coterminal angles of $-5200°$

① $\frac{-5200}{360} = 14.44$ ③ $\theta = 200 + n(360°)$

② $-5200 + 360(15)$
$= 200°$

9. Find the radius of a circle if an arc of 4cm subtends an angle of $30°$ on the circle.

$a = R \times \theta$
$4cm = R \times \frac{\pi}{6}$
$\frac{24}{\pi}$ = RADIUS

10. What is the length of an arc subtended from the sector angle $\pi/5$ if the circle has a radius of 20cm?

$a = \theta \times R$
$a = \frac{\pi}{5} \times 20$
$a = 4\pi$

11. What is the length of the radius of a circle with an arc length of 13.1 cm subtended from a sector of $42°$?

$a = \theta \times R$
$13.1 = 42°\left(\frac{\pi}{180}\right) \times R$
$\frac{13.1 \times 180}{42 \times \pi} = R$
$R = 17.87\ cm$

12. As the time changes from 1:00pm to 3:45pm on a clock, determine the change in radians of the minute hand. Determine the change in radians for the hour hand.

① Hour Hand

1:00pm → 3:45pm.

2 Hours 45 min

= 165 min

② $\theta = \frac{165\text{ min}}{12 \times 60} \times 2\pi$

$= 0.2291\overline{6} \times 2\pi$

$= 1.4398966$ RAD

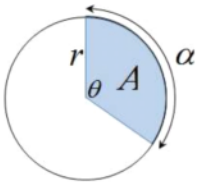
② Minute Hand.

- Each hour → 1 Full rotation 2π
- 45 min → $\frac{3}{4}$ rotation $2\pi \times \frac{3}{4} = \frac{3\pi}{2}$

So 2h. 45 min $= 2\pi + 2\pi + \frac{3\pi}{2}$

$= \frac{11\pi}{2}$ RAD.

13. Derive a formula for the area, A, of a sector of a circle with radius "r", formed by an angle of θ radians. Derive a similar formula when the measure of the angle is in degrees



$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}$

$2\pi \times A = \theta \times \pi r^2$

$A = \frac{\theta \times r^2}{2}$

14. If arc "a" is 6π cm long and the central $\theta = 72°$ then what is the area of the sector "A"?

① Radius:

$a = r \times \theta$

$6\pi = r \times \frac{72 \times \pi}{180}$

$\frac{180 \times 6}{72} = r = 15$ cm

② $A = \frac{72}{360} \times \pi r^2$

$A = \frac{1}{5} \times \pi (15)^2$

$= 45\pi$ cm^2

15. Find the radius of a circle if an arc of 3 subtends an angle of $30°$ on the circle

$3 = \frac{\pi}{6} \times r$

$\frac{18}{\pi}$ = Radius

16. Find the angle in degrees if an arc length of 5cm has a radius of 6cm.

$5\text{ cm} = \theta \times 6\text{ cm}$

$\frac{5}{6}$ radians $= \theta$

17. When an object is moving in a circle, its "angular velocity" is the angle per unit time through which it rotates about the center. A car tire has diameter 64cm. Determine its angular velocity, in radians per second, when the car is travelling at 100km/h.

① Radius $= 32$ cm $C = 64\pi$ cm

② Speed $= \frac{100\text{ km}}{\text{h}} \times \frac{1\text{h}}{3600\text{s}} \times \frac{100{,}000\text{ cm}}{1\text{ km}}$

$= 2777.\overline{77}$ cm/s.

③ Rotations/s $= 2777.\overline{77} \div 64\pi$

$= 13.8155$ rotations/s

④ Angular Velocity $= 13.8155332 \times 2\pi$

$= 86.805$ rad/s

18. What is the smallest angle formed by the x-axis and the line through the points (2,1) and (-8,7)

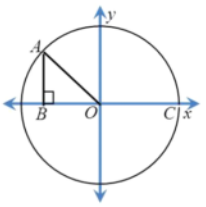
19. What is the sum of: $\sin^2(10°)+\sin^2(20°)+\sin^2(30°)+....+\sin^2(170°)$

a) 1 b) 3 c) 5 d) 9 e) 10

20. In the sequence below, each angle is in radians. What is the largest number of consecutive terms of this sequence that can be positive?

$$\cos x, \cos(x+1), \cos(x+2), \cos(x+3), \cos(x+4), \cos(x+5), \cos(x+6)$$

21. Use a geometric approach with an unit circle to show that for any obtuse angle θ, $\sin\theta = \sin(\pi-\theta)$



22. Use the same approach above to show that for any obtuse angle θ, $\cos\theta = -\cos(\pi-\theta)$

23. Challenge: Use the figure below to prove that $\sin(a-b) = \sin a\cos b - \sin b\cos a$. Hint: Use the "Sine Law" if necessary.