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Name:____________________________ Date:______________________

Math 9 Enriched: Section 1.1 Introduction to Function Notations

1. Given the functions, $f(x)=3x^2-2x$ and $g(x)=-\frac{3x}{2}+3$, find the indicated values:

   i) $f(3)\times g(4)$ ii) $2f(-2)-3g(2)$ iii) $4f(2)\times g(-3)$

2. Given the functions, $f(x)=\sqrt{x}+3$ and $g(x)=2x^2-1$, find the indicated values:

   i) $f(g(x))$

   $f(2x^2-1)=\sqrt{2x^2-1}+3$

   ii) $g(f(x))$

   $g(\sqrt{x}+3)=2(\sqrt{x}+3)^2-1$

   $=2(\sqrt{x}+3)(\sqrt{x}+3)-1$

   $=2(x+6\sqrt{x}+9)-1$

   $=2x+12\sqrt{x}+17$

   iii) $g(f(g(x)))$

   $g(\sqrt{2x^2-1}+3)=2(\sqrt{2x^2-1}+3)^2-1$

   $=2(2x^2-1+6\sqrt{2x^2-1}+9)-1$

   $=4x^2-2+12\sqrt{2x^2-1}+17$

   $=4x^2+12\sqrt{2x^2-1}+29$

   iv) $f(g(3))$

   $g(3)=2(3)^2-1$

   $=17$

   $f(17)=\sqrt{17}+3$

   v) $g(f(18))$

   $f(18)=\sqrt{18}+3$

   $=3\sqrt{2}+3$

   $g(3\sqrt{2}+3)=2(3\sqrt{2}+3)^2-1$

   vi) $g(f(g(5)))$

   $g(5)=49$

   $f(49)=10$

   $g(10)=199$

   vii) $f(f(x))$

   $f(\sqrt{x}+3)=\sqrt{\sqrt{x}+3}+3$

   viii) $g(g(f(25)))$ ix) $f(g(f(50)))$

3. If $f(x)=x^2+3x-10$, find the value of "x" that will make the expression true:

   i) $f(x)=0$

   $x^2+3x-10=0$

   $(x+5)(x-2)=0$

   $x=-5,\ x=2$

   v) $f(x)=8$

   $x^2+3x-10=8$

   $x^2+3x-18=0$

   $(x+6)(x-3)=0$

   $x=-6\ \ x=3$

   vi) $f(x)=-6$

   $x^2+3x-10=-6$

   $x^2+3x-4=0$

   $(x+4)(x-1)=0$

   $x=-4,\ x=1$

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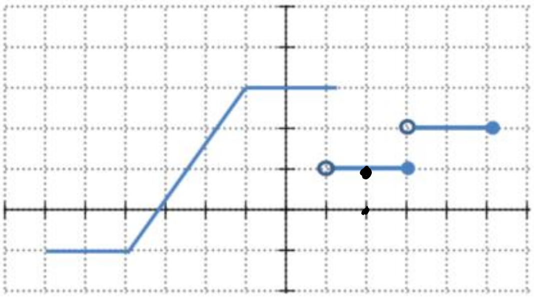
4. Given the graph of $f(x)$, find the indicated values:



$i)\ f(2)=1$

$ii)\ f(1)=3$

$iii)\ f(4)=2$

$v)\ f(?)=3$

$\boxed{-1 \le ? \le 1}$ //

$vi)\ f(-4)\times f(3)$

$-1 \times 1 = -1$ //

5. If $f(x)=x^2-x+2$, $g(x)=ax+b$, and $f(g(x))=9x^2-3x+2$, determine all possible ordered pairs (a,b) which satisfy this relationship.

$f(ax+b) = (ax+b)^2 - (ax+b)+2 = 9x^2-3x+2$

$= \boxed{a^2x^2} + \underline{2abx} + b^2 - \underline{ax} - b + 2 = \boxed{9x^2} - \underline{3x} + 2$

① $a^2 = 9$

$a = \pm 3.$

② $2ab - a = -3.$

$a(2b-1) = -3.$

$2b-1=1$ $2b-1=0.$

$b=0$ $\cancel{b=\frac{1}{2}}$

③ $b^2-b+2=2.$

$b^2-b=0$

$b=0$ $\cancel{b=1}$

$\therefore a=-3,\ b=0.$

6. If $f(x)=2x-1$, determine all real values of "x" such that $(f(x))^2-3f(x)+2=0$

$(2x-1)^2-3(2x-1)+2=0$

$4x^2-4x+1-6x+3+2=0$

$4x^2-10x+6=0$

$2x^2-5x+3=0.$

$\begin{matrix} 2 & -3 \\ 1 & -1 \end{matrix}$

$(2x-3)(x-1)=0$

$\boxed{x=\frac{3}{2} \quad x=1.}$ //

7. A function $f(x)$ has the following three properties:

i) $f(1)=1$,   ii) $f(2x)=4f(x)+6$,   iii) $f(x+2)=f(x)+12x+12$

Calculate the value of $f(6)$.

① $f(2) = 4(f(1))+6$

$= 4+6$

$f(2) = 10$

Don't need this //

② $f(3) = f(1) + 12(1) + 12$

$f(3) = 25$

③ $f(6) = 4f(3) + 6$

$= 4(25)+6$

$= 106$ //

8. Give an example of a function $g(x)$ such that the identity below is true for all values of "x" and "y"

$g(x+y)=g(x)+g(y)$

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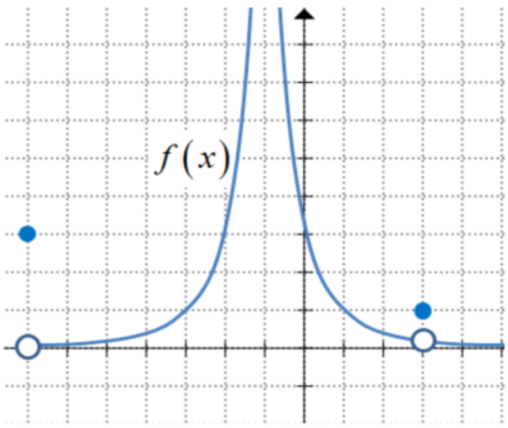
9. Given the graph of $f(x)$, find the value of the following values:



$i)\ f(f(3)) =$

$f(3) = 1$

$f(1) = 1$ //

$ii)\ f(f(0)) =$

$f(0) = 3$

$f(3) = 1$ //

$iii)\ f(-1)$

UNDEFINED //

$v)\ f(-7)\times f(0)$

$3 \times 3 = 9$ //

$vi)\ f(f(x)) = x \quad x = ?$

INVERSE! $f(x) = y$

$f(y) = x$.

ONLY WHEN $x = y$ $\therefore$ $\boxed{x = 1}$ //

$vi)\ f(f(f(-2)))$

$f(-2) = 3$

$f(3) = 1$

$f(1) = 1$ //

10. Let $f(x) = 2^{kx} + 9$, where "k" is a real number. If $f(3) : f(6) = 1:3$, determine the value of $f(9) - f(3)$.

$f(3) = (2^3)^k + 9$

$= 8^k + 9$.

$f(6) = (2^6)^k + 9$

$= 64^k + 9$.

$\dfrac{64^k + 9}{8^k + 9} = 3$.

$64^k + 9 = 3(8^k) + 27$.

$(8^k)^2 - 3(8^k) - 18 = 0$.

$(A - 6)(A + 3) = 0$.

$\therefore 8^k = 6$. not -3 //

$f(9) = (2^9)^k + 9$

$= (8^k)^3 + 9$

$= 225$.

$f(3) = 8^k + 9$

$= 15$ //

$\therefore$

$225 - 15$

$= 210$ //

11. The function $f(x)$ has the property that $f(2x+3) = 2f(x) + 3$ for all values of "x". If $f(0) = 6$, what is the value of $f(9)$?

$f(3) = 2f(0) + 3$

$f(3) = 15$

$f(9) = 2f(3) + 3$

$= 33$ //

12. Given the piece-wise function, what is the value of $f(f(f(3)))$?

$$f(n) = \begin{cases} n^2 & \text{if } n \text{ is even} \\ n+1 & \text{if } n \text{ is odd} \end{cases}$$

① $f(3) = 4$.

② $f(4) = 16$

③ $f(16) = 16^2 = 256$ //

13. The function "f" is defined for integer values only and satisfies the following: //

$$f(n) = \begin{cases} n+2 & \text{if } n < 10 \\ f(n-2) & \text{if } n \geq 10 \end{cases}$$

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14. Let $\phi(x)$ denote the sum of the digits of the positive integer "x". For example, $\phi(8)=8$ and $\phi(123)=1+2+3$. For how many two digit value of "x" is $\phi(\phi(x))=3$?

a) 3 b) 4 c) 6 d) 9 e) 10

$\phi(x)=12 \rightarrow x=93, 39, 48, 84, 57, 75, 66$

$21 \rightarrow$ N/A

$30 \rightarrow$ N/A

$3 \rightarrow x=12, 21, 30$

15. For any three real numbers "a", "b", and "c", with $b \neq c$, the operation ϖ is defined by:

$\varpi(a,b,c)=\dfrac{a}{b-c}$. What is the value of $\varpi(\varpi(1,2,3),\varpi(2,3,1),\varpi(3,1,2))$?

$w(1,2,3) = \frac{1}{2-3} = -1$

$w(2,3,1) = \frac{2}{3-1} = 1$

$w(3,1,2) = \frac{3}{-1} = -3$

$w(-1,1,-3) = \frac{-1}{4} = -1/4$

16. COMC: Let $f(x)=x^2$ and $g(x)=3x-8$,

a. Determine all values of "x" such that $f(g(x))=g(f(x))$

① $f(3x-8)=(3x-8)^2$
$=9x^2-48x+64$

② $g(x^2)=3x^2-8$

③ $9x^2-48x+64=3x^2-8$
$=6x^2-48x+72=0$
$x^2-8x+12=0$

$(x-7)(x-1)=0$
$x=7, x=1$

b. Let $h(x)=3x-r$, determine all values of "r" such that $f(h(2))=h(f(2))$

① $h(2)=6-r$
$f(2)=4$

② $f(6-r)=(6-r)^2$
$=36-12r+r^2$
$h(4)=12-r$

③ $r^2-12r+36=12-r$
$r^2-11r+24=0$
$(r-8)(r-3)=0$
$r=8, r=3$

17. Challenge: Let $f(t)=\dfrac{7^t}{7^t+\sqrt{7}}$. Find the value of the following:

$$f\left(\frac{1}{2014}\right)+f\left(\frac{2}{2014}\right)+f\left(\frac{3}{2014}\right)+\ldots\ldots+f\left(\frac{2013}{2014}\right)$$

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① $f(x)+f(1-x)=\dfrac{7^x}{7^x+\sqrt{7}}+\dfrac{7^{1-x}}{7^{1-x}+\sqrt{7}}$

$(7^x)(7^{1-x})+7^x\sqrt{7}+7^x(7^{1-x})+(7^{1-x})\sqrt{7}$

$$\frac{(7^x)(7^{1-x}) + 7^x\sqrt{7} + 7^x(7^{1-x}) + (7^{1-x})\sqrt{7}}{(7^x)(7^{1-x}) + 7^x(\sqrt{7}) + (7^{1-x})(\sqrt{7}) + 7}$$

$$= \frac{7 + 7^x(\sqrt{7}) + 7 + (7^{1-x})(\sqrt{7})}{7 + 7^x(\sqrt{7}) + 7 + (7^{1-x})\sqrt{7}}$$

$$= 1$$

②
$$\left.\begin{aligned} f\left(\tfrac{1}{2014}\right) + f\left(\tfrac{2013}{2014}\right) &= 1 \\ f\left(\tfrac{2}{2014}\right) + f\left(\tfrac{2012}{2014}\right) &= 1 \\ &\vdots \\ f\left(\tfrac{1006}{2014}\right) + f\left(\tfrac{1008}{2014}\right) &= 1 \end{aligned}\right\} 1006$$

③ $f\left(\frac{1007}{2014}\right) = f\left(\frac{1}{2}\right) = \frac{7^{1/2}}{7^{1/2} + 7^{1/2}} = \frac{1}{2}$

④ $\therefore$ Total $= 1006.5$