\Name:________________________________ Date:____________________

Math Club Logarithmic Functions Worksheet #2

1. Simplify each of the following to a single logarithmic expression:

$\log a + \log b + 2\log c$	$3\log\frac{1}{a} + 2\log a$	$5\log b + \log b^3 - 2\log b$
$3\log_4 a + \log_2 a$	$(\log_a b)(\log_b \sqrt{a})$	$\log_c x^{\log_x \sqrt{c}}$
$a^{\log_a b + \log_a \sqrt{b}}$	$\frac{\log x^7 + \log x^4}{\log x^{12} - \log x^2}$	$\log a - 5\log b + \frac{1}{3}\log c$
$\log_b x^{\log_x a}$	$x^{\log_x 20 - \log_x 4}$	$\frac{\log x^3 + \log x^5}{\log x^6 - \log x^3}$
$\log(1-x^3) - \log(1+x+x^2) - \log(1-x)$	$\frac{1}{\log_a x} + \frac{1}{\log_b x}$	$z^{-3\log_z 5}$

2. Evaluate each of the following (Without a calculator!):

$\log_{10} 10$	$(\log_{10} 1000)\times(\log_{10} 100^2)$	$\log_3 243+\log_9 729$
$\log_2 \sqrt[3]{16}+\log_4 8$	$(\log_6 216)(\log_{64} 16)$	$\log_6 8+\log_6 27$
$6\log_2(1.5)-3\log_2(18)$	$(\log_a b)(\log_b c)(\log_c a)$	$\log_4 20-\log_4 5+\log_4 8$
$\log_2 \sqrt{5}-\log_2 \sqrt{40}$	$\log_4 \sqrt{32}+\log_2 \sqrt[3]{0.5}-\log_2 \frac{1}{8}$	$5^{2\log_{25} 16}$

3. Solve for "x" for each of the following:

$\log_3(\log_8 2)=2$	$\log_5\left(\log_3\left(\log_2 (x-3)^2\right)\right)=0$	$(\log_{10} z)(\log_z y)(\log_y x)=\log_5 25$
$\log_3\left[\log_x\left[\log_2 8\right]\right]=-1$	$2^{x-1}=3^{x-1}$	$(\log_{64} x)(\log_5 16)=2$

4. Solve for "y" $(\log_3 x)(\log_x 2x)(\log_{2x} y) = \log_x x^2$

5. If $\log x^5 y^3 = 25$ and $\log \frac{x}{y} = 3$, then what is the value of $\log x$?

6. Prove the following identity: $\frac{1}{\log_a b} + \frac{1}{\log_c b} = \frac{1}{\log_{ac} b}$

7. Find the value of $\sum_{n=1}^{999} \log \sqrt[3]{\frac{n^2}{n^2+2n+1}}$

8. Simplify the product completely: $\frac{\log_2 3}{\log_4 3} \times \frac{\log_4 5}{\log_6 5} \times \frac{\log_6 7}{\log_8 7} \times \ldots \times \frac{\log_{124} 125}{\log_{126} 125} \times \frac{\log_{126} 127}{\log_{128} 127}$

9. If $\log\left(xy^3\right) = 1$ and $\log\left(x^2 y\right) = 1$. What is $\log(xy)$? Amc 2003 12A

10. If $a \ge b > 1$, what is the largest possible value of $\log_a\left(a/b\right) + \log_b\left(b/a\right)$? AMC 2003 12B

11. How many distinct four-tuples (a, b, c, d) of rational numbers are there with:

    $a\log_{10} 2 + b\log_{10} 3 + c\log_{10} 5 + d\log_{10} 7 = 2005$ Amc 2005 12B

12. The numbers $\log\left(a^3b^7\right),\ \log\left(a^5b^{12}\right),\ and\ \log\left(a^8b^{15}\right)$ are the first three terms of an arithmetic sequence, and the 12th term of the sequence is $\log\left(b^n\right)$. What is the value of "n"? AMC 2008 12A

13. It is given that $\log_6 a + \log_6 b + \log_6 c = 6$, where "a", "b", and "c" are positive integers that form an increasing geometric sequence and $b - a$ is the square of an integer. Find the value of $a + b + c$? AIME2002

14. Challenge: Let $N = \sum_{k=2}^{1000} k\left(\lceil \log_{\sqrt{2}} k \rceil - \lfloor \log_{\sqrt{2}} k \rfloor\right)$ (Here $\lfloor x \rfloor$ is the greatest integer that is less than or equal to "x" ,and $\lceil x \rceil$ is the least integer that is greater than or equal to "x"). Find the remainder when "N" is divided by 1000.

15. For all positive integers “n”, let $f(n) = \log_{2002} n^2$. Let $N = f(11) + f(13) + f(14)$. Which of the folllowing relations is true? AMC 12 2002

A) $N > 1$ B) $N = 1$ C) $1 < N < 2$ D) $N = 2$ E) $N > 2$

1. For all integers “n” greater than 1, define $a_n = \dfrac{1}{\log_n 2002}$. Let $b = a_1 + a_2 + a_3 + a_4$ and $c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14}$. Then what is the value of $b - c$? AMC 12B 2002