Name:___________________________________ Date:_______________________________

Math Contest Club: Complex Variables Worksheet #1

1. Simplify or evaluate the following and express your answer in the form of $a \pm bi$:

$(3+2i)(1-3i)$	$\left(2-\sqrt{-4}\right)+\left(-3+\sqrt{-16}\right)$	$(-1+i)^7$
$\frac{1+2i}{3+4i}+\frac{2i-5}{5i}$	$\sqrt{\frac{-3}{2}}+\sqrt{\frac{-2}{3}}$	$\frac{(2+i)^2}{2-i}+\frac{(2-i)^2}{2+i}$
$\frac{1+i}{1-i}-\frac{1-i}{1+i}$	$\left(\sqrt{9+40i}+\sqrt{9-40i}\right)^2$	$\frac{1+2i}{3-4i}+\frac{2-i}{5i}$

2. Solve the following and express your answer in the form of $a \pm bi$:

$x^3=8$	$(5-2i)-(x+4i)=7-6i$	$x^2=5-12i$
$x^2+(i-5)x+12-5i=0$	$x^2-15+8i=0$	$x^2=-3+4i$
$5x^2+4=0$	$z^2-16-16i\sqrt{3}=0$	$15\sqrt{-144}-\left(3\sqrt{-i}+1\right)^2=7-6i$

3. Find the value of $(-i)^{4n-1}$ when "n" is a negative integer.

4. If $\left(r+\frac{1}{r}\right)^2=3$, then find the numerical value of $r^3+\frac{1}{r^3}$

5. If "z" is a complex number and $\bar{z}$ is its conjugate, then determine the complex numbers which satisfy the equation: $5z^2-4z\left(\bar{z}\right)=(1-3i)z$

6. If "z" is a complex number and $\bar{z}$ is its conjugate, then determine the value of : $z^5-\left(\bar{z}\right)^5$

7. Given that one root is -2, find the other roots of $f(x)=x^3-4x^2-2x+20$

8. Given that the two roots of a rational polynomial are $2-i$ and -3, determine the polynomial of the smallest degree such that $f(0)=4$

9. Given that two roots of the polynomial equation: $x^6 - 6x^5 + 14x^4 - 22x^3 + 25x^2 + 8x = 60$, are $2i \; and \; 2-i$, then what are the real roots?

10. Find the sum of the following: $1 + 2i + 3i^2 + 4i^3 + ... + 1000i^{999} + 1001i^{1000}$

11. Given the following equation, find the value of "k" if "k" and "m" are integers:

$$\left[2-\left(-2+i\sqrt{3}\right)-\left(-2-i\sqrt{3}\right)\right]\left[2+\left(-2+i\sqrt{3}\right)^2+\left(-2-i\sqrt{3}\right)^2\right]\left[2-\left(-2+i\sqrt{3}\right)^4-\left(-2-i\sqrt{3}\right)^4\right]=2^k3^m$$

AMC 12B 2004

16. A function f is defined by $f(z) = i\overline{z}$, where $i = \sqrt{-1}$ and $\overline{z}$ is the complex conjugate of z. How many values of z satisfy both $|z| = 5$ and $f(z) = z$?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 8

AMC 12A 2007

18. The polynomial $f(x) = x^4 + ax^3 + bx^2 + cx + d$ has real coefficients, and $f(2i) = f(2+i) = 0$. What is $a+b+c+d$?

(A) 0 **(B)** 1 **(C)** 4 **(D)** 9 **(E)** 16

AMC 12B 2008

19. A function f is defined by $f(z) = (4+i)z^2 + \alpha z + \gamma$ for all complex numbers z, where α and γ are complex numbers and $i^2 = -1$. Suppose that $f(1)$ and $f(i)$ are both real. What is the smallest possible value of $|\alpha| + |\gamma|$?

(A) 1 **(B)** $\sqrt{2}$ **(C)** 2 **(D)** $2\sqrt{2}$ **(E)** 4

AMC12B 2005

22. A sequence of complex numbers $z_0, z_1, z_2, \ldots$ is defined by the rule

$$z_{n+1} = \frac{iz_n}{\overline{z_n}},$$

where $\overline{z_n}$ is the complex conjugate of z_n and $i^2 = -1$. Suppose that $|z_0| = 1$ and $z_{2005} = 1$. How many possible values are there for z_0?

(A) 1 **(B)** 2 **(C)** 4 **(D)** 2005 **(E)** 2^{2005}

AMC 12A 2002

24. Find the number of ordered pairs of real numbers (a, b) such that $(a+bi)^{2002} = a - bi$.

(A) 1001 **(B)** 1002 **(C)** 2001 **(D)** 2002 **(E)** 2004

AMC12A 2008

23. The solutions of the equation $z^4 + 4z^3i - 6z^2 - 4zi - i = 0$ are the vertices of a convex polygon in the complex plane. What is the area of the polygon?

(A) $2^{\frac{5}{8}}$ **(B)** $2^{\frac{3}{4}}$ **(C)** 2 **(D)** $2^{\frac{5}{4}}$ **(E)** $2^{\frac{3}{2}}$