Name:____________________________ Date:__________________________

Math 10 Enriched: Section 5.5 Solving and Graphing Systems of Inequalites

1. Solve each inequality using a number line. Remember to check for extraneous roots and asymptotes:

a) $\sqrt{2x-1}<9$	b) $3\sqrt{3x-1}\le 12$
c) $\sqrt{2x+7}-9>0$	d) $-7+9\sqrt{2x+3}\ge 9$
e) $2\sqrt{2x+3}>x+3$	f) $\sqrt{8x+5}<\sqrt{2x+2}$

g) $\frac{36}{x} > -3$	h) $\frac{6}{x+9} \le 2$
i) $\frac{3x-4}{x+4} < 7$	j) $\frac{6}{x+3} < x+8$
k) $\frac{4x+3}{4x-1} \le 2x+4$	l) $\frac{-4}{x+2} \le x+7$

m) $\sqrt{x+2} > x$	n) $\sqrt{x+2} \le \frac{1}{x+2}$
o) $\sqrt{x+2} < \frac{x}{x+2}$	p) $\sqrt{4x-12} \le \sqrt{2x+12}$
q) $\sqrt{-3x} - \sqrt{x+4} \ge 0$	r) $\frac{1}{x} > \frac{3}{x+4}$

2. Solve each inequality. Check for extraneous roots. Draw your solution on a number line:

a) $\lvert x+3\rvert+\lvert x-6\rvert<16$	b) $\lvert 2x+1\rvert+\lvert 3x-5\rvert<18$
c) $\lvert 2x+3\rvert+\lvert 3x-8\rvert>15$	d) $\lvert x+3\rvert+\lvert 5-x\rvert<16$
e) $\lvert 2x+1\rvert+\lvert 4-3x\rvert>18$	f) $\lvert 2x+3\rvert+\lvert 4-3x\rvert>15$

g) $\frac{8}{x-3}+\frac{8}{x+1}<-3$	h) $\frac{12}{x-6}+\frac{6}{2x-1}\geq -1$

3. Solve the following inequality. Draw your solution on a number line: $|x+1|-2=-|x-3|+2$

4. Solve the following inequality. Draw your solution on a number line: $\frac{4}{x+3}+\frac{4}{2x-4}\leq x-2$

5. Solve the following inequality. Draw your solution on a number line: $2x-\frac{9}{2x}<-\frac{5}{2}$

6. What are all values of “x” that satisfy the inequality? $\sqrt{x^2-3x+2}<x+3$ CNML 2-5

7. What is the ordered triple of positive integers (a,b,c) with “a” as small as possible, for which $|ax+b|\le c$ is equivalent to $\frac{-10}{3}\le x\le 1$?

8. What are all values of “t” for which the inequality is satisfied for all real values of “x”? $\frac{x^2-tx-2}{x^2-3x+4}>-1$ CNML 6-6 2001

9. Challenge: Prove that following statement is true for any positive integer “n”: $\sum_{k=1}^{n}\frac{k}{k^4+k^2+1}<\frac{1}{2}$ UBCMath Circles 2012

1. Solve the inequality:. (3 marks)

Solve for x: $4 + |2x + 3| < 5$.

of the four circles?

2-5. What are all values of x that satisfy $\sqrt{x^2 - 3x + 2} < x + 3$?

4-4. What is the ordered triple of positive integers (a, b, c), with a as small as possible, for which $\|ax + b\| \le c$ is equivalent to $-\frac{10}{3} \le x \le 1$?	4-4.

6-6. What are all values of t for which the inequality $\frac{x^2 - tx - 2}{x^2 - 3x + 4} > -1$ is satisfied for all real values of x?	6-6.

Problem 6-6

Since $f(x) = x^2 - 3x + 4$ has a negative discriminant, f doesn't change sign. In fact, f is always positive, so if we multiply by f, we do not change the direction of the inequality. Thus, $x^2 - tx - 2 > -x^2 + 3x - 4$. Collecting terms, we see that $2x^2 - (t+3)x + 2 > 0$. This condition is true for *all* real values of x whenever the discriminant of the left side is a negative number. Finally, $(-t-3)^2 - 16 < 0 \Leftrightarrow t^2 + 6t - 7 < 0 \Leftrightarrow \boxed{-7 < t < 1}$.

3-6. What are all real values of p for which the inequality

$$-3 < \frac{x^2 + px - 2}{x^2 - x + 1} < 2$$

is satisfied by all real values of x?

Problem 3-6

Method I: Since $f(x) = x^2-x+1$ has a negative discriminant, its graph will not cross the x-axis. Since $f(0)$ is 1, f is positive, so we may multiply through by f to get the equivalent inequality $-3x^2+3x-3 < x^2+px-2 < 2x^2-2x+2$. The left-hand inequality is equivalent to $4x^2+(p-3)x+1 > 0$. The right-hand inequality is equivalent to $x^2-(p+2)x+4 > 0$. These two inequalities hold for *all* real x if and only if the discriminants of both quadratics are negative; that is, if $(p-3)^2-16 < 0$ and $(p+2)^2-16 < 0$. Equivalently, $|p-3| < 4$ and $|p+2| < 4$. Combine the inequalities $-1 < p < 7$ and $-6 < p < 2$ to get the result $\boxed{-1 < p < 2}$.

Method II: This solution is Method I, but we won't use the discriminant. Since $x^2-x+1 = (x-\frac{1}{2})^2+\frac{3}{4}$, $f(x) > 0$. Next, $y = 4x^2+(p-3)x+1$ is a parabola with its vertex a minimum at $x = \frac{3-p}{8}$. Thus, $4x^2+(p-3)x+1 > 0$ or $(p-3)^2-16 < 0$. The minimum point of $y = x^2-(p+2)x+4$ is at $x = \frac{p+2}{2}$; and if $x^2-(p+2)x+4 > 0$, then $(p+2)^2-16 < 0$.

